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# MECHANISM OF FATIGUE-CRACK GROWTH UNDER COMPRESSIVE EXTERNAL STRESSES

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*A specimen with a crack shaped like a thin long defect with parallel edges and a rounded tip of finite radius is considered. When the specimen is subjected to compressive cyclic loads, the stresses in zones adjacent to the crack tip vary according to a sign-variable cycle because of plastic strains occurring at the crack tip. This can cause fatigue-crack growth. Results of numerical simulations and experimental data support the possibility of fatigue-crack growth in the field of compressive external stresses.*

**Key words:** *fatigue crack, compression, plastic zone.*

**1. Formulation of the Problem.** In the classical fracture mechanics, compressive stresses are assumed to have no effect on the character of failure and fatigue-crack growth rate [1–3]. It is believed that, if a loading cycle contains compressive stresses ($\sigma_{\min} < 0$), the edges come in contact and the crack is fully closed during compression (when the external load $\sigma_0$ tends to the value of $\sigma_{\min}$). In this case, it is common practice to analyze the fracture process within a pulsed loading cycle from the stress $\sigma_0 = 0$ to the tensile stress $\sigma_0 = \sigma_{\max}$. This implies that, regardless of the magnitude of $\sigma_0 = \sigma_{\min}$, the processes of fatigue-damage accumulation are the same for a constant value of $\sigma_{\max}$. However, the results of experimental studies of a fatigue crack subjected to compressive stresses [4, 5] contradict the classical concepts of fracture mechanics.

In particular, the effect of compressive-stress amplitude on fatigue-crack growth was studied [4]. In the experiments, steel specimens were loaded in such a manner that the maximum cyclic stresses $\sigma_{\max}$ were positive and identical for the entire batch of the specimens tested. The minimum cyclic stresses ($\sigma_{\min}$) were compressive. The minimum stresses differed from one specimen of the batch to another. The experiments showed that the crack growth rate, first, is independent of the crack length and, second, depends substantially on the magnitude of compressive stresses: the higher the absolute value of $\sigma_{\min}$, the faster the crack grows.

These results differ from the classical concepts according to which the crack growth rate increases with the crack length [1, 2] and compressive-stress amplitude has no effect on the crack growth since the crack is closed upon compression [1].

Experimental studies of fatigue-crack growth in steel specimens subjected to varying compressive stresses showed the following [5].

1. In a notched specimen, a crack is initiated at those points of the notch where the maximum compressive stresses occur. The crack has a plane front and always propagates in the direction perpendicular to the external load.

2. The fatigue-crack growth rate increases with the amplitude of the loading cycle.

It should be noted that the problem is of practical importance. In some engineering structures such as railway rails, cracks are initiated under the action of sign-variable, mostly compressive stresses. It is therefore important to establish the reasons for crack initiation and propagation mechanism.

**2. Basic Hypotheses and Assumptions.** 1. A plane problem is considered. The specimen has an infinite size in the plane and a unit thickness. The material is continuous and isotropic.

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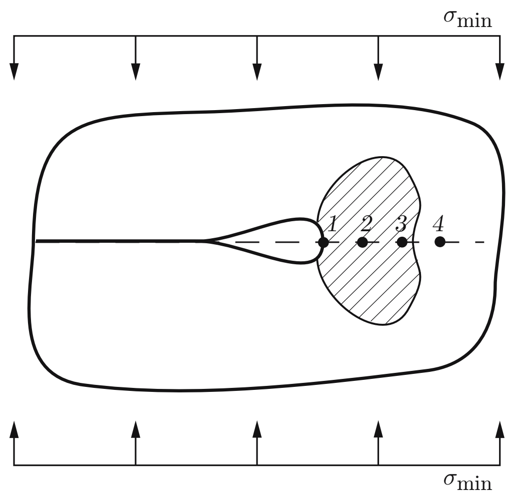


Fig. 1. Tip of a thin defect under compressive loading (the plastic-strain region is hatched).

2. The specimen contains a long defect shaped like a through cut, crack, etc. The edges of the defect are parallel ant its tip is rounded.

3. The material of the structure is plastic and described by a curve with a linear hardening segment. In the tensile-stress zone, the deformation curve is limited by the stresses $\sigma_z$, which is the actual stress at the moment of rupture. In the compressive-stress zone, the curve is unlimited and its lower branch tends to infinity. Since plastic strains occur at the crack tip, we consider the deformation curve of the material in the strain intensity ($\varepsilon_i$) — stress intensity ($\sigma_i$) axes.

4. The material of the specimen belongs to the class of cyclically stable materials. This means that its mechanical characteristics remain unchanged under repeated-varying loads. The tensile and compressive elastic moduli and yield points are identical. The Bauschinger effect is ignored. It is worth noting that most structural materials are cyclically hardened materials [6, 7]. For these materials, the yield point increases under rigid cyclic loading.

5. The specimen is subjected to varying external compressive stresses applied at infinity. It is assumed that the maximum stresses of the cycle are equal to zero. This assumption makes it easier to understand the crack-growth process without violating the generality of reasoning. The minimum stresses of the cycle are lower than the yield point.

6. The failure criterion is as follows: failure of the material occurs if the stress intensity exceeds the value of $\sigma_z$, which is the rupture strength. Compression does not cause the failure of the material.

**3. Fatigue-Crack Growth Mechanism.** We consider a specimen under external cyclic compressive loading. As a loading parameter, we use the stresses $\sigma_0$ applied at infinity.

At the loading stage of the cycle, the stress varies from $\sigma_0 = 0$ to $\sigma_0 = \sigma_{\min}$. As the magnitude of the external stresses increases, the edges of the crack approach each other and, as a result, the crack is closed but not over its entire length [8]. Since the crack tip is rounded, there is a zone near the crack tip where the edges are not in contact. We consider the crack at the moment when $\sigma_0 = \sigma_{\min}$ (Fig. 1). The following key points should be noted. The main body of the material works in the elastic region. The zone of negative plastic strains has a very small size, as compared to the specimen size, and is immediately adjacent to the crack tip. It is essential that the higher the magnitude of $\sigma_{\min}$, the more fully the crack is closed, the larger the plastic zone, and the higher the plastic strains. But, in any case, the crack is never closed fully.

We consider the deformation curve of the material (Fig. 2). At the moment the loading parameter reaches $\sigma_0 = \sigma_{\min}$, points 1, 2, 3, and 4 shown in Fig. 1 correspond to points on the negative branch of the curve. In this case, the strains at each point correspond to the values of $\varepsilon_{(1)}$, $\varepsilon_{(2)}$, $\varepsilon_{(3)}$, and $\varepsilon_{(4)}$.

At the unloading stage of the cycle, the stress varies from $\sigma_0 = \sigma_{\min}$ to $\sigma_0 = 0$. When the load is maximal ($\sigma_0 = \sigma_{\min}$), almost the entire specimen works in the elastic region except for a very small region near the crack tip. Therefore, after the external stresses are removed, the specimen is restored to its original size. For points lying

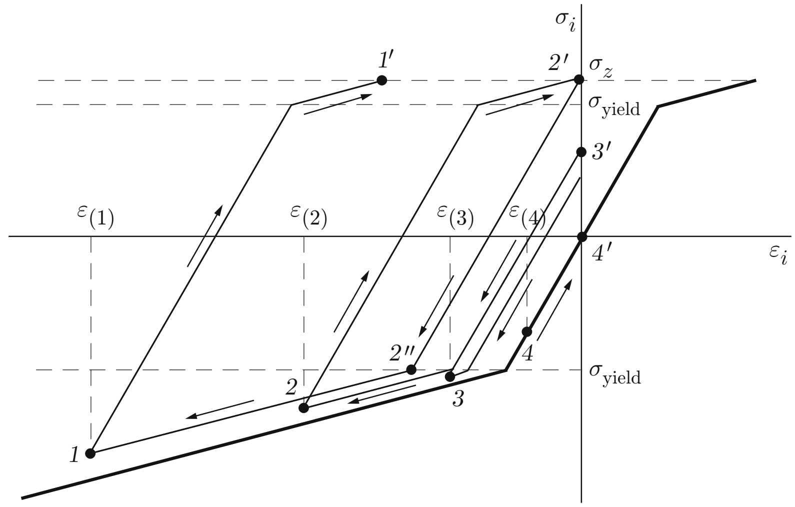


Fig. 2. Variation in the stress–strain state of the region adjacent to the defect tip for cyclic compressive loading by external stresses.

inside the plastic zone, this loading is stiff. Consequently, at the end of the unloading stage of the cycle, when the external stresses reach $\sigma_0 = 0$, the strains at all points of the specimen should be approximately equal to those at the first stage but should have the opposite sign.

We consider the unloading process using the deformation curve (see Fig. 2). At the end of the unloading stage, the strain at point 1 should be approximately equal to $|\varepsilon_{(1)}|$, i.e., the point is unloaded to the state $\varepsilon_i \approx 0$. However, the case is possible where the stress at the defect tip attains the value of $\sigma_z$ during unloading (point $1'$ in Fig. 2), whereas the loading parameter $\sigma_0$ has not yet reached zero. Further unloading results in failure of the material at the tip of the defect. Obviously, there is a point inside the plastic zone (point 2 in Fig. 1) at which the stress and strain intensities become $\sigma_i = \sigma_z$ and $\varepsilon_i = 0$, respectively, at the end of unloading (point $2'$ in Fig. 2). Since the stress intensity does not exceed the value of $\sigma_z$, no failure occurs at this point. Consequently, the material of the specimen located between points 1 and 2 (Fig. 1) fails and, as a result, the crack length increases from point 1 to point 2 during unloading. The newly formed tip of the crack is rounded. The reason is that, first, the crack tip is loaded by tensile stresses at the end of the unloading stage (point $2'$ in Fig. 2) and, second, even if it is assumed that the material works in the elastic region up to its failure, a $1/\sqrt{\rho}$ singularity [2] ($\rho$ is the rounding radius at the crack tip) occurs at the defect tip. If $\rho = 0$ (the crack tip is pointed), the stresses and, hence, strains tend to infinity. This results in blunting of the crack tip. Consequently, the stresses drop to finite values. Owing to the plastic properties of the material, the rounding radius at the crack tip increases substantially (which leads to a further decrease in stresses at the tip), and deformation becomes irreversible.

At points located farther than point 2 from the crack tip (Fig. 1), no failure of the material occurs. After unloading, the strains at these points are close to zero and, in the $\sigma_i$–$\varepsilon_i$ diagram, these points are located on the $\sigma_i$ axis (for example, point $3'$ in Fig. 2). After unloading, point 4 and all other points lying outside the plastic zone have the coordinates $\sigma_i = 0$ and $\varepsilon_i = 0$.

Figure 3 shows the crack tip under cyclic loading. The solid line shows the crack at the end of the loading stage ($\sigma_0 = \sigma_{\min}$). A plastic zone is formed at the crack tip. The dashed line shows the crack at the end of unloading ($\sigma_0 = 0$). In this case, the plastic zone is reduced and the material inside this zone is in tension. In the next loading cycle, when the crack tip reaches point 2 (Fig. 1), the loading following line $2'$–$2''$ (Fig. 2) and then the hardening segment to point 1. The cycle is closed. At the unloading stage of the previous cycle, the crack extended from point 1 to point 2 by the distance $\Delta l$. Then, at the loading stage of the next cycle, the plastic zone is shifted in the direction of crack propagation and restores the size it had at the end of the loading stage of the previous loading cycle. The dotted line in Fig. 3 shows the crack tip at the end of the loading stage of the next cycle.

**4. Specific Features of Deformation at the Tip of a Thin Cut under Cyclic Compressive Loading.** To support the ideas explaining crack growth under cyclic compressive loading, a numerical experiment was performed. For this purpose, deformation of a $400 \times 200$ mm rectangular plate with a thin cut 200 mm long

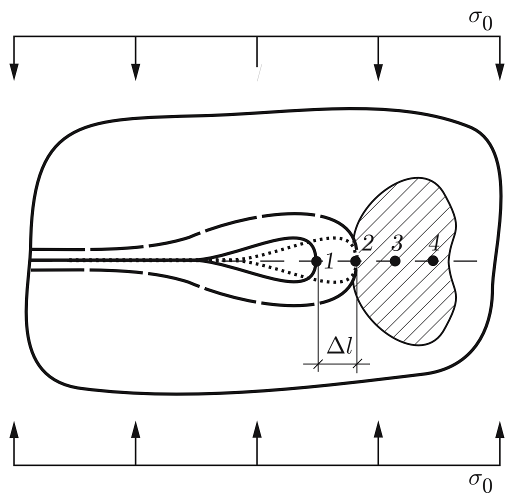


Fig. 3. Crack tip under cyclic compressive loading: the solid line shows the crack at the end of the loading stage of the cycle ($\sigma_0 = \sigma_{\min}$), the dashed line shows the crack at the end of the unloading stage of the cycle ($\sigma_0 = \sigma_{\max}$), and the dotted line shows the crack at the end of the loading stage of the next cycle ($\sigma_0 = \sigma_{\min}$).

and 20 $\mu$m wide located symmetrically in the plate was considered. The edges of the defect were parallel and its tip was modeled by a circumference of radius 10 $\mu$m. The plate was subjected to cyclic compressive stresses $\sigma_0$ perpendicular to the cut. The elastic modulus of the material was 200 GPa, Poisson's ratio was 0.28, the yield point was 250 MPa, and the stress-strain curve had a linear hardening segment with a tangent modulus of 200 MPa. The problem was solved by the finite-element method. Along the tip contour, 20 nodes were located. The plane stress state of the plate was considered. The loading was performed according to the following law. The external load $\sigma_0$ increased linearly from zero and reached the value of ($-50$ MPa) in 10 sec. During the next 10 sec, the absolute value of the load decreased to zero. In this way, one loading cycle was modeled. The elastoplastic problem was solved by the method of small elastoplastic strains.

We consider some calculation results. For $\sigma_0 = -50$ MPa, the stress $\sigma_y \approx -310$ MPa occurs at the crack tip (Fig. 4). The plastic-zone size calculated using the Mises criterion does not exceed 11 mm. Note that the crack is not fully closed. Small fragments of the defect extended for approximately 2 mm from the cut tip remain open. At the moment the plate is completely unloaded, the stresses at the crack tip change the sign and reach the value $\sigma_y = +351$ MPa (Fig 5). It is worth noting that, for pulsed loading of the specimen, the stresses at the cut tip follow a nearly symmetric cycle, where the positive component slightly prevails and the amplitude exceeds the magnitude of the external load $\sigma_0$ by a factor of 6 to 7.

**5. Experimental Verification.** To verify experimentally the possibility of initiation and growth of cracks under external compressive stresses, specimens made of a D16 aluminum alloy (with the yield point of 290 MPa and ultimate strength of 440 MPa) and annealed rail steel (with the yield point of 520 MPa and ultimate strength of 720 MPa) were tested. The specimens were shaped like rectangular plates 10 mm thick with thin lateral initiating notches. The loading was performed by a GRM-1M pulsator with a frequency of 10 Hz. For aluminum specimens, the minimum and maximum cyclic stresses calculated as average stresses in the effective cross section were $\sigma_{\min} \approx -200$ MPa and $\sigma_{\max} \approx -20$ MPa, respectively. For steel specimens, the loading cycle parameters were $\sigma_{\min} \approx -375$ MPa and $\sigma_{\max} \approx -65$ MPa. In all cases, fatigue cracks started from the tips of the crack initiators and grew perpendicularly to the direction of compressive loading.

The first 50,000 loading cycles applied to the aluminum specimen can be considered as low-cycle fatigue. After approximately 5000 cycles, the main cracks (2 in Fig. 6) started from the pointed tips of the initiating notches (1 in Fig. 6) and, propagating perpendicularly to the external load, became 5 mm long after 144,000 cycles. After 100,000 cycles, peripheral cracks (3 in Fig. 6) were initiated on the specimen surface at a distance from the initiating notches. The location and directions of the peripheral cracks are such that they are seemingly enclose the main crack, which started earlier. Further loading of the specimen was characterized by retardation of the main-crack

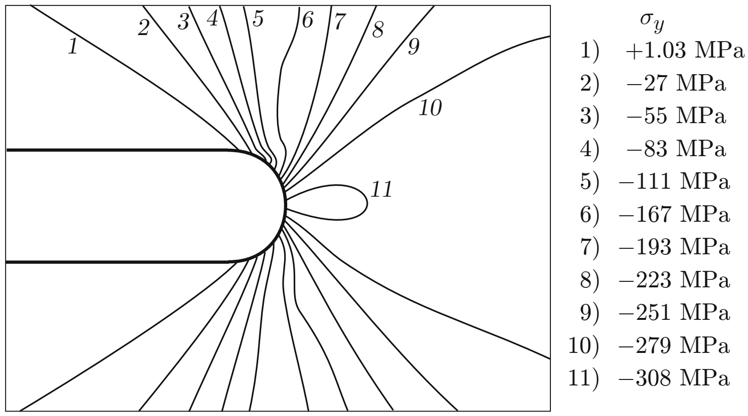


Fig. 4. Stress field $\sigma_y$ in the vicinity of the tip of a thin cut at the moment of the maximum loading of the specimen by the external compressive load ($\sigma_0 = -50$ MPa).

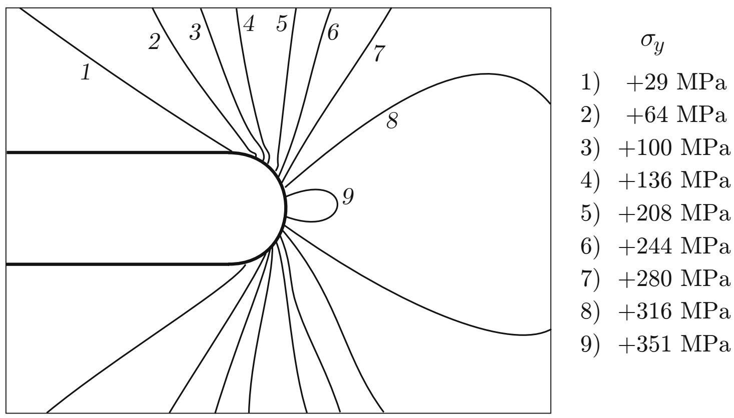


Fig. 5. Residual stress field $\sigma_y$ in the vicinity of the tip of a thin cut at the moment of complete unloading of the specimen ($\sigma_0 = 0$).

growth and by intense development of a system of peripheral cracks. They were also initiated in regions that deformed elastically in the beginning of experiment. The experimental data suggest that most of the peripheral cracks are through cracks, which pierce the structure from the front surface to the back surface. Figure 6 shows the surface of the aluminum specimen after 450,000 loading cycles.

**6. Conclusions.** The model for fatigue-crack initiation and growth considered above explains well the experimental data on crack growth under compressive stresses [4, 5]. The major findings can be summarized as follows.

1. A fatigue crack in the compressive-stress field always grows along the trajectory perpendicular to the direction of the maximum compressive stresses. It is at these points of the defect in a loaded specimen that the plastic strains reach the maximum absolute values. Therefore, the most favorable conditions for initiation and growth of cracks are formed here. To support this statement, we can cite the data of [5]. Considering crack-starter notches of various shapes, the authors infer that the crack either starts immediately to grow perpendicularly to the direction of the minimum principal stresses ($\sigma_3$) or, if the notch plane does not coincide with the minimum-stress plane, the crack turns until its front becomes plane and is oriented perpendicularly to the loading direction. Thus, in most cases it is impossible to make a fatigue crack extend according to the $K_{\rm II}$, $K_{\rm III}$, or mixed mode fracture mechanism.

2. The fatigue-crack growth rate is proportional to the amplitude of the loading cycle. Indeed, the crack growth rate is proportional to the length of crack extension in one loading cycle ($\Delta l$ in Fig. 3). For a constant maximum stress of the loading cycle, this length depends on the intensity of plastic strains at the defect tip, which,

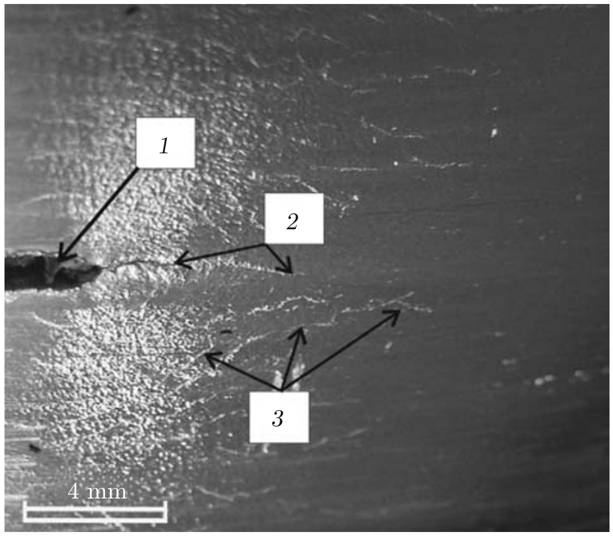


Fig. 6. Surface of an aluminum specimen after 450,000 loading cycles:
1) crack initiator; 2) main crack; 3) peripheral cracks.

in turn, depends on the value of compressive stresses, i.e., amplitude of the loading cycle. This fact was supported experimentally [4, 5].

3. There exists a certain value of compressive stresses $\sigma_0$ (or amplitude of a cycle) below which a fatigue crack does not grow at all. If the minimum stresses of the loading cycle are such that the strain $\varepsilon$ at the point located at the crack tip is within the interval $0 \leqslant |\varepsilon| \leqslant |\varepsilon_{(2)}|$ (see Fig. 2), the tensile stresses at the end of the unloading stage of the cycle do not exceed $\sigma_Z$ and, hence, the crack does not grow.

4. The fatigue-crack growth rate does not depend on the crack length (see [4]). As the plastic zone is formed in a specimen under compression, the magnitude of plastic strains for a crack that is not fully closed is mainly determined by the open zone of the defect whose size is independent of the total crack length.

5. To analyze the fatigue-crack growth mechanism in a specimen under sign-variable cyclic loading, one should take into account the effect of the negative component.

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